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Real Estate Mathematics

Basic Formulas and Functions Real Estate Applications

Answer Key is on page 427

BASIC FORMULAS AND FUNCTIONS

Adding and multiplying fractions
Converting decimals and percentages
Converting fractions and percentages
Multiplying percentages
Calculating area

Adding and multiplying fractions

Adding

1. Formulas:

Same denominator: $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Different denominator: $\frac{a}{c} + \frac{b}{d} = \frac{ad+bc}{cd}$

2. Examples: $\frac{2}{5} + \frac{6}{5} = \frac{8}{5}$

$$\frac{3}{4} + \frac{4}{7} = \frac{(3 \times 7) + (4 \times 4)}{(4 \times 7)} = \frac{37}{28}$$

Problem 1: $\frac{3}{19} + \frac{2}{7} = ?$

Multiplying

1. Formula: $\frac{a}{c} \times \frac{b}{d} = \frac{ab}{cd}$

2. Example: $\frac{4}{9} \times \frac{2}{3} = \frac{8}{27}$

Problem 2: $\frac{4}{14} \times \frac{3}{8} = ?$

Converting decimals and percentages

Converting a decimal to a percent

1. Formula: (decimal number) $\times$ 100 = percent number

2. Examples: $.473 \times 100 = 47.3\%$
 $3.456 \times 100 = 345.6\%$
 $.0042 \times 100 = .42\%$

Problem 3: Convert the following decimals to percentages:

$$2.65 = \%$$

$$0.294 = \%$$

$$0.005 = \%$$

Converting a percent to a decimal

1. Formula: $\frac{\text{percent number}}{100} = \text{decimal number}$

2. Examples: $\frac{47.3\%}{100} = .473$

$$\frac{345.6\%}{100} = 3.456$$

Problem 4: Convert the following percentages to decimals:

$$72.1\% =$$

$$90.2\% =$$

$$5.79\% =$$

Converting fractions and percentages

Converting a fraction to a percent

1. Formulas:

$$(1) \quad \frac{a}{b} = a \text{ divided by } b = \text{decimal number}$$

$$(2) \quad \text{decimal number} \times 100 = \text{percent number}$$

2. Examples: $\frac{4}{5} = 4 \text{ divided by } 5 = 0.8 = 80\%$

$$\frac{9}{3} = 9 \text{ divided by } 3 = 3.0 = 300\%$$

Problem 5: Convert the following fractions to percentages:

$$\frac{8}{9} = \quad \%$$

$$\frac{3}{6} = \quad \%$$

$$\frac{14}{42} = \quad \%$$

Converting a percent to a fraction and reducing it

1. Formula: $X\% = \frac{X}{100}$

$$\frac{X \div a}{100 \div a} = \text{reduced fraction}$$

where "a" is the largest number that divides *evenly* into numerator and denominator. If unknown, try 2, 3, 5, or 7.

2. Example $45\% = \frac{45}{100} = \frac{45 \div 5}{100 \div 5} = \frac{9}{20}$

Problem 6: Convert and reduce the following percentages:

$$40\% =$$

$$16\% =$$

Multiplying Percentages

1. Formulas:

- (1) convert percent to decimal by dividing by 100
- (2) whole amount $\times$ decimal = partial amount

2. Example: 33% of 400

- (1) 33% divided by 100 = .33
- (2) 400 $\times$.33 = 132

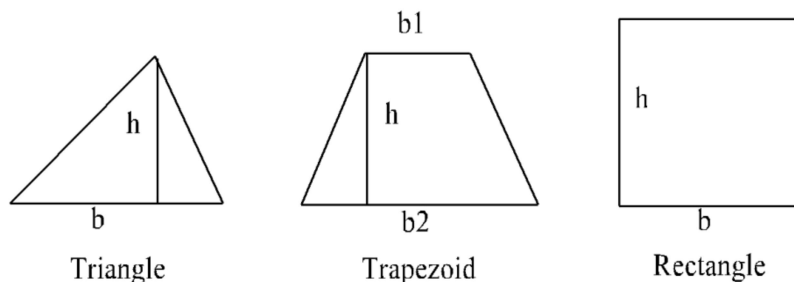
Problem 7: What is 75% of 280?

Calculating area

To find the area of an irregular shape, try dividing it into triangles, rectangles, squares or trapezoids and calculate the areas of those parts; the area of the whole shape is then the sum of the areas of all of its subparts.

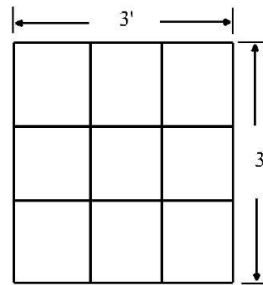
Base and height

1. Formulas for area of three- and four-sided shapes use a product of base and height. In the formulas, a represents area, b represents base, h represents height, SF represents square feet.
2. The base of a triangle, square, or rectangle may be any side; a trapezoid has two bases, its two parallel sides.
3. The height of a triangle is the length of a perpendicular line from the base to the triangle's opposite point. Height in a square, rectangle or trapezoid is the length of a line which is perpendicular to the base line(s).



Area of a square or rectangle

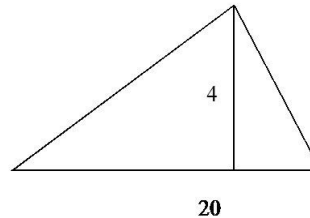
1. Formula: area (a) = base (b) x height (h) $a = b \times h$
2. Example: A square measures 3 feet on each side.



Its area is: $3 \times 3 = 9 \text{ SF}$

Area of a triangle

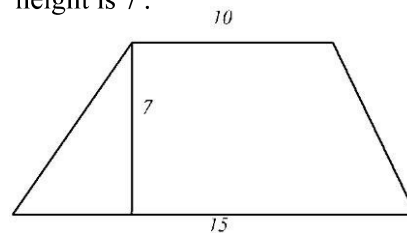
1. Formula: $a = \frac{b \times h}{2}$
2. Example: A triangle has a 20' base and a 4' height.



Its area is: $a = \frac{20 \times 4}{2} = 40 \text{ SF}$

Area of a trapezoid

1. Formula: $a = \frac{a (b_1 + b_2)}{2}$
2. Example: A trapezoid's two bases are 10' and 15', and its height is 7'.



Its area is: $a = \frac{7 (10 + 15)}{2} = 87.5 \text{ SF}$

Problem 8: Calculate the areas of the following shapes:

a rectangle with height of 4' and base of 36'

a =

a square with a side of 16'

a =

a trapezoid with height of 5' and bases of 6' and 8'

a =

REAL ESTATE APPLICATIONS

Legal descriptions (Chapter 9)

Listing agreements (Chapter 12)

Brokerage business (Chapter 13)

Sales contracts (Chapter 14)

Appraisal (Chapter 16)

Finance (Chapter 17)

Investments (Chapter 18)

Taxation (Chapter 19)

Closings (Chapter 21)

Legal descriptions (Chapter 9)

Linear measures

(cm = centimeter; m = meter; km = kilometer)

1 inch	=	1/12 foot	=	1/36 yard
1 foot	=	12 inches	=	1/3 yard
1 yard	=	36 inches	=	3 feet
1 rod	=	16.5 feet	=	1/320 mile
1 mile	=	5280 feet	=	1760 yards = 320 rods

1 centimeter	=	1/100th m	
1 meter	=	100 cm	= 1/1000th km
1 kilometer	=	1,000 m	

Area measures

1 square inch	=	1/144th square foot
1 square foot	=	1/9th square yard
1 square yard	=	9 square feet

1 acre	=	1/640 sq. mi	=	43,560 SF	=	208.71 ft x 208.71 ft
1 square mile	=	640 acres	=	1 section	=	1/36 township
1 section	=	mile x 1 mile	=	640 acres	=	1/36 township
1 township	=	6 mi x 6 mi	=	36 sq. mi.	=	36 sections

Metric conversions

(cm = centimeter; m = meter; km = kilometer)

1 inch	=	2.54 cm		
1 foot	=	30.48 cm	=	.3048 m
1 yard	=	91.44 cm	=	.9144 m
1 mile	=	1609.3 m	=	1.60 km
1 centimeter	=	.3937 inch		
1 meter	=	39.37 inches	=	3.28 feet = 1.094 yards
1 kilometer	=	3,281.5 feet	=	.621 mile

Fractions of sections, acres, and linear dimensions

Fraction	# Acres	Feet X Feet
1 section	640 acres	5280 X 5280
1/2 section	320 acres	5280 X 2640
1/4 section	160 acres	2640 X 2640
1/8 section	80 acres	2640 X 1320
1/16 section	40 acres	1320 X 1320
1/32 section	20 acres	660 X 1320
1/64 section	10 acres	660 X 660

Calculating area from the legal description

1. Formula:

- (1) First multiply all the denominators of the fractions in the legal description together
- (2) Then divide 640 by the resulting product.

2. Examples:

N 1/2 of the SW 1/4 of Section 6:

$$\frac{640}{(2 \times 4)} = \frac{640}{8} = 80 \text{ acres}$$

W 1/2 of the NW 1/4 of the NE 1/4 of Section 8

$$\frac{640}{(2 \times 4 \times 4)} = \frac{640}{32} = 20 \text{ acres}$$

Problem 9: Calculate the acreage of the following:

SW 1/4 of the N 1/2 of the E 1/2 of Section 14

SE 1/4 of the NW 1/4 of the SE 1/4 of Section 20

Listing agreements (Chapter 12)

Co-brokerage commission

1. Formulas: $\text{sale price} \times \text{commission rate} = \text{total commission}$
 $\text{total commission} \times \text{split rate} = \text{co-brokerage commission}$
2. Example: A house sells for \$600,000. The commission is 6%, and the co-brokerage split is 50-50.
 $\$600,000 \times 6\% = \$36,000$ total commission $\times 50\% =$
\$18,000 co-broker's commission

Agent's commission

1. Formula: $\text{broker's commission} \times \text{agent's split rate} = \text{agent's commission}$
2. Example: Assume an \$18,000 broker's commission and a 60% - 40% agent-broker split rate.
 $\$18,000 \times .6 = \$10,800$ agent's commission (\$7,200 to broker)

Problem 10: A property is co-brokered by listing broker Schroeder and selling broker Hobson for \$425,000. The co-brokerage split is 50-50. Schroeder's agent, Joachim, is on a 65% split schedule. Hobson's selling agent, Wallace, splits 50-50 with her broker. If the total commission rate is 7%, what are the participants' commissions?

Broker Schroeder:	\$
Broker Hobson:	\$
Schroeder's agent, Joachim:	\$
Hobson's agent, Wallace:	\$

Brokerage business (Chapter 13)

Goodwill calculation

1. Formula: Goodwill = Price - Value of assets
2. Example: A seller wants \$1 million for a business. Assets in the business, including inventory, furniture, equipment, leasehold improvements, and working capital, have a total value of \$750,000. The goodwill is:

$$\$1,000,000 - 750,000 = \$250,000 \text{ goodwill}$$

Problem 11: A prospective purchaser complained to a seller that the selling price had far too much goodwill: \$200,000. After all, the assets only totaled \$352,000. What was the price?

Sales contracts (Chapter 14)

"Percentage of listing price" calculation

1. Formula: Percentage of listing price = offer divided by listing price
2. Example: A property listed for \$400,000 receives an offer for \$360,000. The percentage of listing price is:

$$\$360,000 \text{ divided by } 400,000 = 90\%$$

Problem 12: A seller receives an offer of \$674,000 on a property listed at \$749,000. How much is the offer as a percent of the listing price?

Earnest money deposit calculation

1. Formula: Deposit = Listing price x required percentage
2. Example: A seller requires a 2% deposit on a property listed for \$320,000. The required deposit is:

$$\$320,000 \times 2\% = \$6,400$$

Problem 13: A seller requires a 1.5% deposit on all offers. A buyer wants to offer \$312,000 for the property. What must the deposit be?

Rent escalations

1. Formula: $\text{New rent} = \text{current rent} \times (100\% + \text{escalation rate})$
2. Example: An apartment's rent is scheduled to increase by 6%. If the current rent is \$1800, the new rent is:
$$\$1800 \times (100\% + 6\%) = \$1,800 \times 106\% = \$1,908$$

Problem 14: A tenant's rent is currently \$650 per month. This rent is scheduled to increase 5% per year. What will the tenant's rent be at the beginning of the **third** year from now?

FIRPTA withholding

1. Formula: $\text{FIRPTA withholding} = \text{gross proceeds from sale} \times 15\%$
2. Example: Gross proceeds on a FIRPTA-regulated property sale are \$340,000. The required withholding amount is:
$$\$340,000 \times 15\% = \$51,000$$

Appraisal (Chapter 16)

Adjusting comparables

1. Rules:
 - a. NEVER adjust the subject!
 - b. If the comparable is better than the subject, subtract value from the comparable
 - c. If the comparable is **worse** than the subject, **add** value to the comparable
2. Examples:
 - a. A comparable has a pool and the subject does not. The appraiser estimates the value contribution to be \$25,000. Adjust the comparable by entering -25,000 in the CMA.
 - b. A comparable has 3 bedrooms and the subject as 4. The appraiser estimates the value contribution of a bedroom to be \$15,000. Adjust the comparable by entering +15,000 in the CMA.

Problem 15: Identify the proper adjustments for the following:

- (a) The subject has a two-car garage, while the comparable does not. Value of garage is \$33,000.

Adjustment:

- (b) A comparable has a fireplace, and the subject does not. Value of fireplace is \$8,000.

Adjustment:

- (c) The subject has 1,500 square feet. The comparable has 1,600 square feet. The value of extra square feet is \$200/SF.

Adjustment:

Comparative Market Analysis (CMA)

- 1. Steps:
 - a. Identify comparable sales
 - b. Compare comparables to the subject and make adjustments to comparables.
 - c. Weight values indicated by adjusted comparables for the final value estimate of the subject.

Problem 16: Adjust and weigh comparables to indicate a subject value

- a) Using the data and assumptions provided below, complete the worksheet by filling in value adjustments for each blank entry.
- b) Total the values of each comparable's adjustments to derive a net value change for the comparable.
- c) Reconcile the comparables by weighting their values against each other and estimating a value for the subject.
- d) Explain your reasoning.

Comparables data

Subject:	3 bedrooms, two baths, kitchen, living room, family room; 2,500 square feet of gross living area; 3-car attached garage; condition is good.
Comparable A:	Sold for \$850,000; located in subject's neighborhood with similar locational advantages; house approximately same age as subject; lot size similar to subject; condition similar to subject; two bedrooms, one bath; 2,200 square feet of gross living area.
Comparable B:	Sold for \$950,000; located in subject's neighborhood with similar locational advantages; house ten years newer than subject; lot size considerably larger than the subject's; better condition than subject; four bedrooms, three baths; gross living area is similar.
Comparable C:	Sold for \$1,000,000; located in subject's neighborhood with similar locational advantages; house eight years older than subject; lot size similar to subject; condition similar to subject; three bedrooms, three baths; 2,500 square feet of gross living area.
Comparable D:	Sold for 1,090,000; located in a more desirable neighborhood than subject; house approximately same age and size and condition as subject, with two bedrooms, and two baths; 1,900 square feet of gross living area.

Adjustment Assumptions

Location:	no value differences
Financing & terms:	no value differences
Lot size:	\$10,000 value difference for Comparable B
# Bedrooms and bathrooms:	\$10,000 per room
Effective age and condition:	poor -- \$20,000; good – no change; excellent -- \$20,000
Gross living area:	\$10,000 per 100 feet

CMA Adjustment Worksheet

	Subject	A	B	C	D
Sale price	???	850,000	950,000	1,000,000	1,090,000
Location		\$ _____	\$ _____	\$ _____	\$ _____
Age		\$ _____	\$ _____	\$ _____	\$ _____
Lot size		\$ _____	\$ _____	\$ _____	\$ _____
Condition		\$ _____	\$ _____	\$ _____	\$ _____
Bedrooms	3	\$ _____	\$ _____	\$ _____	\$ _____
Baths	2	\$ _____	\$ _____	\$ _____	\$ _____
Gross living area	2,500 SF	\$ _____	\$ _____	\$ _____	\$ _____
Net adjustments		\$ _____	\$ _____	\$ _____	\$ _____
Indicated value	???	\$ _____	\$ _____	\$ _____	\$ _____

Rationale for comp adjustments:

Comments on weighting of adjustments and estimated value of the subject:

Problem 16 Answers

Completed CMA Adjustment Worksheet

	Subject	A	B	C	D
Sale price		850,000	950,000	1,000,000	1,090,000
Location		equal	equal	equal	-20,000
Age		equal	-20,000	+20,000	equal
Lot size		equal	-10,000	equal	equal
Condition	good	equal	-20,000	equal	equal
Bedrooms	3	+10,000	-10,000	equal	+10,000
Baths	2	+10,000	-10,000	equal	equal
Gross living area	2,500	+30,000	equal	equal	+60,000
Net adjustments		+50,000	-70,000	+20,000	+50,000
Indicated value	1,000,000	900,000	880,000	1,020,000	1,140,000

Discussion of Weighting, Reconciliation and Value Estimate

The completed table indicates four estimated values for the subject, ranging from \$880,000 to 1,140,000.

For comparable A, the estimator has made three adjustments -- for the number of bedrooms and baths, and for gross living area -- totaling \$50,000, or 5.9% of the purchase price.

For comparable B, the estimator has deducted values for age, lot, condition, bedrooms, and baths. The total of five adjustments amounts to \$70,000, or 7.4% of the sale price.

For comparable C, the estimator has adjusted only for the age. The adjustment of \$20,000, 2% of sale price, raises the indicated value of the subject to \$1,020,000.

For comparable D, a deduction has been made for the comparable's superior location. This is offset by additions for the number of bedrooms and gross living area reflecting where the comparable is inferior to the subject. The three adjustments amount to a total of +\$50,000, or 4.6% of the sale price.

Weighting the indicated values of the comparables, the estimator has developed an indicated value of \$1,000,000 for the subject. Underlying this conclusion is the fact that Comparable C is by far the best indicator of value, and therefore receives the heaviest weighting as a value indicator. This property has the fewest adjustments (1) and smallest total adjustment amount (2%). Comparable D might be the second-best indicator, since it has the second-lowest number of adjustments (3), and the net adjustment is also second-best as a percent of the sale price (4.6%). Comparable A might be the third best indicator, since it has the second smallest number of total adjustments (3) but the second highest net adjustment percent (5.9%). Comparable B has the greatest number of adjustments (5) and the highest percent of sale price (7.4%) and is therefore the least reliable indicator.

Income capitalization

Gross rent multiplier (GRM)

1. Formula: gross rent x multiplier = value
2. Example: \$2,000 x 135 GRM = \$270,000

Gross income multiplier (GIM)

1. Formula: gross annual income x GIM = value
2. Example: \$45,000 x 9 GIM = \$405,000

Net income capitalization

1. Formula: annual net operating income (NOI) ÷ capitalization rate = value
2. Example: \$50,000 ÷ .10 = \$500,000

Problem 17: An apartment building grosses \$450,000 annually, nets 350,000, and has a capitalization rate of 9%. Prevailing GIMs are 8.

(a) What is the property value using the GIM?

(b) What is the property value using net income capitalization?

Interest only loans

1. Formulas: interest payment (I) = principal (P) x interest rate (R)
annual interest payment ÷ 12 = monthly interest payment
monthly interest payment x 12 = annual interest payment

$$I = P \times R$$

$$R = \frac{I}{P}$$

$$P = \frac{I}{R}$$

2. Examples: A \$300,000 interest-only loan @ 10% has annual payments of \$30,000 and monthly payments of \$2,500.

$$\text{Annual interest} = \$300,000 \times 10\% = \$30,000$$

$$\text{Monthly interest} = \$30,000 \div 12 = \$2,500$$

The loan amount of an interest-only loan that has an annual interest rate of 8% and a monthly interest payment of \$700 is \$105,000.

$$\text{Annual interest} = \$700 \times 12 = \$8,400$$

$$\text{Loan amount} = \$8,400 \div .08 = \$105,000$$

Problem 18:

- (a) A \$250,000 loan carries a 7% rate. What is the monthly interest payment?
- (b) A \$300,000 loan has monthly payments of \$2,000. What is its annual interest rate?
- (c) A 12% loan has annual payments of \$15,000. What is the loan amount?

Loan-to-value (LTV) ratio

1. Formulas: $\text{loan amount} = \text{market value} \times \text{LTV}$
 $\text{LTV} = \text{loan amount} \div \text{market value}$
2. Example: A 75% LTV will allow a lender to make a loan of \$375,000 on a \$500,000 property.
 $\text{loan amount} = \$500,000 \times 75\% = \$375,000$
 $\text{LTV} = \$375,000 \div \$500,000 = 75\%$

Problem 19:

- (a) A lender requires \$90,000 down on a \$400,000 property. Calculate the lender's required LTV.
- (b) A property is valued at \$600,000. The lender will allow a maximum LTV of 75%. How much can the buyer borrow on the property?

Income underwriting ratio calculation

1. Formulas: Conventional:
 $\text{monthly PITI} = (25\text{-}28\%) \times \text{monthly gross income}$
FHA:
 $\text{monthly PITI} = 31\% \times \text{monthly gross income}$
2. Examples: A borrower has monthly gross income of \$2,000. Conventional lenders are using a ratio of 28%. The borrower can afford the following monthly PITI payments:
Conventional:
 $\text{PITI} = 28\% \times \$2,000 = \$560$
FHA:
 $\text{PITI} = 31\% \times \$2,000 = \$620$

Problem 20: A borrower earns \$4,000/month and pays \$600/month in debt repayments. A conventional lender requires a 26% income ratio, and an FHA lender requires 31%. What monthly PITI can this person afford based on the income ratio?

Conventional: \$

FHA: \$

Debt underwriting ratio calculation

1. Formulas: Conventional:

Expense = (36% x gross income) – monthly debt

FHA:

Expense = (43% x gross income) - monthly debt

2. Example: An individual has a monthly gross income of \$6,000, and has monthly debt payments of \$900. The borrower can afford the following monthly housing expense:

Conventional:

Expense = (36% x \$6,000) - 900 = \$1,260

FHA:

Expense = (43% x \$6,000) – 900 = \$1,680

Problem 21: A borrower earns \$4,000/month and makes monthly debt payments of \$600. What monthly payment for housing can this person afford based on the debt ratio?

Conventional: \$

FHA: \$

Points

1. Formula: 1 point = 1% (.01) of loan amount
2. Example: A lender charges 3 points (3%) on a \$350,000 loan. The points charges are:

$$3 \text{ points} = 3\%$$
$$.03 \times \$350,000 = \$10,500$$

Problem 22: A lender is charging 2.75 points on a \$240,000 loan. How much must the borrower pay for points?

Investments (Chapter 18)

Appreciation

1. Formulas: $\text{total appreciation} = \text{current value} - \text{original price}$
 $\text{total appreciation rate} = \text{total appreciation} / \text{original price}$
 $\text{one-year appreciation rate} = \text{one-year appreciation} / \text{prior-year value}$
2. Example: A house bought for \$500,000 appreciates \$50,000 each Year for 3 years.

$$\text{total appreciation} = \$650,000 - \$500,000 = \$150,000$$
$$\text{total appreciation rate} = \$150,000 \div \$500,000 = 30\%$$
$$\text{first-year rate} = \$50,000 \div \$500,000 = 10\%$$

Problem 23: A property is purchased for \$360,000. A year later it is Sold for \$410,000. What is the amount of appreciation, and what is the appreciation rate?

Equity

1. Formula: $\text{equity} = \text{current value} - \text{current loan amount}$
2. Example: A buyer bought a property for \$600,000 with a loan of \$450,000. The house has appreciated \$60,000 and the buyer has reduced the original loan by \$30,000. The buyer's current equity is:
$$\text{Equity} = (\$600,000 + 60,000) - (\$450,000 - 30,000) = \$240,000$$

Problem 24: A property is purchased for \$450,000 with a \$75,000 downpayment. Five years later the property is worth \$540,000, and the loan balance has dropped \$12,500. What is the owner's new equity?

Pre-tax cash flow

1. Formula and example:

potential rental income	\$50,000
- vacancy and collection loss	3,000
= effective rental income	47,000
+ other income	2,000
= gross operating income (GOI)	49,000
- operating expenses	20,000
- reserves	3,000
= net operating income (NOI)	26,000
- debt service	15,000
= pre-tax cash flow	11,000

Problem 25: An apartment building has a potential income of \$300,000 and vacancy of \$12,000. Its bills total \$128,000, and \$12,000 has been reserved for repairs. Payments on the loan total \$88,000. What is the property's pre-tax cash flow?

Tax liability

1. Formula
and example:

net operating income (NOI)	26,000
+ reserves	3,000
- interest expense	15,000
- cost recovery expense	5,000
= taxable income	9,000
x tax rate (28%)	
= tax liability	2,520

Problem 26: The property from the previous problem has annual cost recovery of \$28,000. Out of the annual debt service, \$8,000 is non-interest principal payback. The property owner's tax rate is 28%. What is the property's annual tax?

Annual depreciation (cost recovery) expense

1. Calculation:
 - a. identify improvements-to-land ratio
 - b. identify value of improvements: ratio x property price
 - c. divide value of improvements by total depreciation term
2. Example: A property was bought for \$400,000. 75% of the value is allocated to the improvement. The property falls in the 39-year depreciation category.
 - (1) improvements-to-land ratio = 3:1, or 75%
 - (2) improvement value = \$400,000 x 75% = \$300,000
 - (3) annual depreciation = \$300,000 ÷ 39 = \$7,692

Problem 27: A property is purchased for \$400,000. Improvements account for 80% of the value. Given a 39-year depreciation term, what is the annual depreciation expense?

Capital gain

1. Formula
and example: (residential property)

Selling price of property	\$300,000
- Selling costs	24,000
= Amount realized (ending basis)	\$276,000
Beginning basis (price) of property	\$250,000
+ Capital improvements	10,000
- Total depreciation expense	0
= Adjusted basis of property	260,000
Amount realized (ending basis)	\$276,000
- Adjusted basis of property	260,000
= Capital gain	\$ 16,000

Problem 28: A principal residence is bought for \$360,000. A new tile roof is added, costing \$15,000. Five years later the home sells for \$440,000, and the closing costs \$35,000. What is the homeowner's capital gain?

Return, rate of return, and investment amount

1. Formulas:

$$\frac{\text{net operating income}}{\text{price}} = \text{return on investment (ROI)}$$

$$\frac{\text{cash flow}}{\text{cash invested}} = \text{cash-on-cash return (C on C)}$$

$$\frac{\text{cash flow}}{\text{equity}} = \text{return on equity (ROE)}$$

2. Example: A property is bought for \$200,000 with a \$50,000 down payment and a \$150,000 interest-only loan. The property has a net income of \$20,000 and a cash flow of \$8,000. In addition, the property has appreciated \$30,000.

$$\begin{aligned}\text{ROI} &= \$20,000 \div \$200,000 = 10\% \\ \text{C on C} &= \$8,000 \div \$50,000 = 16\% \\ \text{ROE} &= \$8,000 \div \$80,000 = 10\%\end{aligned}$$

Problem 29: A multi-unit rental property was bought four years ago for \$1,200,000 with a \$200,000 down payment. The property now rents for \$8,500 per month. Expenses and debt service are \$1,000/month and \$6,500/month respectively. An appraiser estimates the property's current value at \$1,450,000. The investor pays off her principal balance at a rate of \$5,000 per year. Compute the following investment returns for the investor:

ROI =

C on C =

ROE =

Taxation (Chapter 19)

Tax rate calculation

1. Formula: $\text{tax rate (millage rate)} = \frac{\text{tax requirement}}{\text{tax base}}$

2. Example: A municipality has a revenue requirement of \$10,000,000 after accounting for its revenues from sale of utilities. This requirement has to be covered by property tax. The real estate tax base, after homestead exemptions, is \$300,000,000. The tax rate will be:

$$\frac{10,000,000}{300,000,000} = .0333 \text{ 33.33 mills}$$

Problem 30: Barrington has an annual budget of \$25,000,000 to be paid by property taxes. Assessed valuations are \$300,000,000, and exemptions total \$25,000,000. What must the tax rate be to finance the budget?

Homestead exemption calculation

1. Formula and example

assessed value	\$360,000
- <u>homestead exemption</u>	<u>50,000</u>
taxable value	\$310,000

Taxing the property

1. Formulas: taxable value of property x tax rate (mill rate) for each taxing authority in jurisdiction

total tax = sum of all taxes by taxing authority
2. Example: The taxable value of a property after exemptions is 400,000 and tax rates are as shown. The property's tax bill will be:

School tax:	\$400,000	x 10 mills	=	\$4,000
City tax:	\$400,000	x 4 mills	=	1600
County tax:	\$400,000	x 3 mills	=	1200
Total tax:				\$6,800

Problem 31: A homeowner's assessed valuation is \$225,000. The homestead exemption is \$25,000. Tax rates for the property are 8 mills for schools; 3 mills for the city; 2.5 mills for the county; and .5 mills for the local community college. What is the homeowner's tax bill?

Special assessments calculation

1. Formula:
 - a. Identify total costs to be assessed
 - b. Calculate prorated share for each property impacted
 - c. Multiply cost x prorated share
2. Example: A canal will be dredged at a cost of \$200,000. The improvement affects 30 properties with a total canal frontage of 4,000 feet. One property has 200' of frontage. Its assessment bill will be:

(1) $200' \div 4,000' = 5\%$ share

(2) $\$200,000 \times 5\% = \$10,000$ assessment

Problem 32: A street beautification project is to cost \$25,000. The project affects 20 properties having a total of 2,000 front feet. One owner's lot has 75 front feet. What will this owner's special assessment be?

Closings (Chapter 21)

Prorations

1. Formulas
and rules:

Accounting for common items paid (or received) in advance vs arrears

	arrears	advance	debit	credit
real estate taxes	x		seller	buyer
rents received by seller		x	seller	buyer
utilities	x		seller	buyer

Whose share is charged to whom?

- if buyer pays taxes in arrears: charge seller, credit buyer for seller's portion
- if seller received rents in advance: charge seller, credit buyer for buyer's portion
- if seller pays utilities in advance: credit seller, charge buyer for seller's portion

Calculating the proration: 360-day method and 365-day method

1. Calculate the daily proration amount

- a. 360-day method: divide the annual amount by 360 or monthly amount by 30
- b. 365-day method: divide the annual proration amount by 365, or monthly amount by # days in that month

2. Calculate # of seller's days

- a. 360-day method: use 30 days for each month; actual number of seller days within the month, counting (or not counting) the day of closing
- b. 365-day method: use actual number of days for each month and partial month

3. Calculate the seller's share

Multiply the daily amount times the number of seller's days – both methods

4. Calculate buyer's share (both methods)

Subtract seller's share (from #3) from total share

2. **Example:** A rental property closes on January 25 and the closing day is the seller's. The 365-day method will be used for the prorations. Monthly rent already received by seller is \$2,400. Annual real estate taxes to be paid in arrears by buyer are \$4,000. Round to the nearest cent.

1. **Rent proration:** (monthly; 365-day method)

Daily amount: $\$2,400 \text{ monthly rent} \div 31 \text{ days in January} = \$77.42$

Seller's days: 25

Seller's share: $\$77.42 \times 25 = \$1,935.50$

Buyer's share: $\$2,400 - \$1,935.50 = \$464.50$

Credit buyer and debit seller for buyer's share of \$464.50

2. **Tax proration:** (annual; 365-day method)

Daily amount: $\$4,000 \div 365 = \$10.96$

Seller's days: 25

Seller's share: $\$10.96 \times 25 = \$274$

Buyer's share: $\$4,000 - \$274 = \$3,726$

Credit buyer and debit seller for seller's share of \$274

Problem 33: A rental property closes on March 15th. Proratable income and expenses are: rental income of \$1,800/month, received in advance by seller, March 1; annual taxes of \$4,800/year, to be paid in arrears by buyer, January 1 of the year after sale. The day of closing is the seller's. February has 28 days. Prorate the items using the 365-day method, and assign debits and credits.

rent:	seller's share	\$
	buyer's share	\$
	debit seller/credit buyer	\$
	debit buyer/credit seller	\$
taxes:	seller's share	\$
	buyer's share	\$
	debit seller/credit buyer	\$
	debit buyer/credit seller	\$

REAL ESTATE MATHEMATICS: ANSWER KEY

Problem 1:	$\frac{3}{19} + \frac{2}{7} = \frac{21+38}{133} = \frac{59}{133}$	Problem 23:	\$50,000; 13.89%
Problem 2:	$\frac{4}{14} \times \frac{3}{8} = \frac{12}{112}$	Problem 24:	\$177,500
Problem 3:	2.65 = 265% 0.294 = 29.4% 0.005 = .5%	Problem 25:	\$60,000
Problem 4:	72.1% = .721 90.2% = .902 5.79% = .0579	Problem 26:	\$14,560
Problem 5:	8/9 = 88.8% 3/6 = 50% 14/42 = 33.3%	Problem 27:	\$8,205
Problem 6:	40% = 40/100 = 2/5 16% = 16/100 = 4/25	Problem 28:	\$30,000
Problem 7:	210	Problem 29:	(a) ROI = 7.5% (b) C on C = 6.0% (c) ROE = 2.55%
Problem 8:	144 SF 256 SF 35 SF	Problem 30:	90.9 mills, or 9.09%
Problem 9:	40 acres 10 acres	Problem 31:	\$2,800
Problem 10:	Schroeder: \$5,206.25 Hobson: \$7,437.50 Joachim \$9,668.75 Wallace: \$7,437.50	Problem 32:	\$937.50
Problem 11:	\$552,000	Problem 33:	rent: seller's share \$870.96 buyer's share \$929.04 debit seller/credit buyer \$929.04
Problem 12:	90%		taxes: seller's share \$973.16 buyer's share 3,826.84 debit seller/credit buyer \$973.16
Problem 13:	\$4,680		
Problem 14:	\$716.63 keep or change?		
Problem 15:	(a) +33,000 to comparable - 8,000 to comparable - 20,000 to comparable		
Problem 16:	\$1,000,000		
Problem 17:	(a) \$3,600,000 (b) \$3,888,888		
Problem 18:	(a) \$1,458 8% \$125,000		
Problem 19:	(a) 77.5% (b) \$450,000		
Problem 20:	Conventional: \$1,040/month FHA & VA: \$1,240/month		
Problem 21:	Conventional: \$840 FHA: \$1,120		
Problem 22:	\$6,600		